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**SOLAR PROTON FLUENCES  
AS OBSERVED DURING 1966-1972  
AND AS PREDICTED FOR 1977-1983  
SPACE MISSIONS**

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**JOSEPH H. KING**



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**— GODDARD SPACE FLIGHT CENTER —  
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SOLAR PROTON FLUENCES AS OBSERVED DURING  
1966 - 1972 AND AS PREDICTED FOR  
1977 - 1983 SPACE MISSIONS

Joseph H. King  
National Space Science Data Center

October 1973

GODDARD SPACE FLIGHT CENTER  
Greenbelt, Maryland 20771

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ABSTRACT

The probability with which any given proton fluence level will be exceeded during a space mission is computed for missions to be flown during the active phase of the next solar cycle (1977-1983). This probability is a function of fluence level, proton energy threshold, and mission duration. Data on the major solar proton events of 1966-1972 are given; it is argued that only this data set (and not that of the previous solar cycle) is appropriate for estimating next-cycle fluences. The most significant feature of the current-cycle data is that the solar proton fluences observed during one week (August 1972) constituted between 69% ( $> 10$  MeV) and 84% ( $> 60$  MeV) of the entire-cycle fluence. Probabilities of such anomalously large events are treated separately from probabilities associated with other events. Probable numbers of each of the two types of events are estimated from Burrell's extension of Poisson statistics. Fluences of all future anomalously large events are assumed to have a common spectrum, that given by the August 1972 event. Fluences of the ordinary events are assumed to obey a log normal distribution. It is shown that for much of the confidence-level mission-duration regime of interest, at least one anomalously large event will occur; and given such an occurrence, the ordinary-event contribution to mission fluence is negligible. This analysis permits the mission planner with a specified mission duration and confidence level to determine how many anomalously-large-event occurrences he must allow for and, if none, to determine how much ordinary-event fluence he must allow for as a function of energy.



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## SECTION 1. INTRODUCTION

Given the quasi-random nature of the occurrences, fluences and spectra of past solar proton events, it is desired to derive, for a space mission of given duration and orbital characteristics flown at a specified epoch in the future, the smallest mission-fluence levels (as a function of energy) whose probabilities of not being exceeded are equal to or greater than some specified confidence level. The purpose of this analysis is to present a statistical model of solar proton fluences relevant to future space missions; dosimetry and shielding considerations are avoided.

The three main areas in this analysis relate to the questions of data, analysis, and results: what is known about past events, how is the available data analyzed to derive probabilities for future mission fluences, and what probabilities are thereby obtained. This analysis will differ from previous analyses in that solar proton fluxes for all major solar-cycle-20 events are now available, and in that the statistical analysis of the data consists of a new combination of the best features of previous analyses, particularly those of Yucker (1972) and of Burrell (1972). (These documents contain references to earlier works of a similar intent.) This paper avoids intercomparing of past analytic techniques.

Section 2 contains a list of event fluences, spectral parameters, and other characteristics for the major solar proton events of solar cycle 20. The most significant aspect of the data is the occurrence,

within a one-week interval (in August 1972) of the seven-year active phase, of most (69-84%) of the cycle-integrated proton fluences. Cycle-20 fluences are contrasted to previously published cycle-19 fluences.

Section 3 contains a discussion of the relevance of past fluences to the future. The variability of solar cycles is addressed, and it is argued that solar cycle 21 is more likely to be similar to the present cycle, 20, than to the past cycle, 19. Given this and the greater confidence one has in the cycle-20 data, this analysis is limited to a consideration of cycle-21 mission-fluence probabilities based on cycle-20 event fluences.

The fourth section presents the technique followed in this analysis. The basic approach is to consider two classes of events, one class populated by ordinary events whose fluence levels are describable by a distribution function, and the other class containing anomalously large events about whose fluence distribution too little is known to permit description by a mathematical function. For cycle 20, the large event of August 1972 is taken to be anomalously large while all other events are considered to be ordinary. Probable numbers of mission-encountered events of each class are computed using Burrell's extension of Poisson statistics.

All anomalously large events are assumed to have the same spectrum as that of the August 1972 event. The fluences of the ordinary events are described by a log normal distribution function. Convolution equations, somewhat similar to those employed by Yucker, are used to

determine probabilities for fluence levels given the occurrence of any specified number of ordinary events. Final probability values require a combination of probabilities of event occurrences and fluences for the two classes of events.

The fifth section contains the results of the calculations. The most important is the finding that if the probability of having no anomalously large event is less than some specified confidence level, for a given mission duration, then it is permissible to neglect the occurrence of ordinary events in determining mission fluence. A similar result was obtained by Burrell. A figure is given identifying the region of the confidence-level mission-duration plane in which ordinary events are negligible. For this region, which covers much of the confidence-level mission-duration regime of interest, a figure is given by which the number of anomalously large events occurring is determined as a function of probability level and mission duration. For the regime in which ordinary events are not negligible, fluences of 30 MeV protons are plotted as a function of probability level and mission duration. A spectral function is also given by which the plotted > 30 MeV fluences may be transformed to other energy thresholds.

The sixth section is a discussion of the results. Probable sources of error are considered; the largest is the lack of a distribution function to describe the fluences of anomalously large events. Galactic proton fluences are contrasted to solar proton fluences.



Extensions of the analysis to spacecraft whose trajectories will involve significant (but not total) magnetospheric shielding or significant time away from a heliocentric distance of 1 A.U. (astronomical unit) are considered.

## SECTION 2. THE DATA

Table 1 contains the basic interplanetary solar proton fluence data for solar cycle 20. The data include instantaneous peak and time-integrated fluxes, an exponential rigidity (or energy for August 1972) spectral parameter, and an indicator of whether solar protons of energies above approximately 500 MeV were observed by the Deep River Neutron Monitor. This list of 25 events includes all periods of about a week in which the time-integrated flux of protons above 10 MeV exceeded  $2.5 \times 10^7/\text{cm}^2$ . This selection of 25 periods includes all (20) periods in which the  $> 30$  MeV proton flux exceeded  $5.0 \times 10^6/\text{cm}^2$  and all (19) periods in which the  $> 60$  MeV flux exceeded  $1.0 \times 10^6/\text{cm}^2$ . That is, the smallest five and six events listed : 30 and 60 MeV are in fact smaller than events not included in Table 1. It will become apparent from the results of this analysis that for missions of reasonable duration and for predictions of reasonable confidence levels this event list is longer than necessary. The main point to note from the data of Table 1 is that the August 1972 fluxes of protons above 10, 30, 60, and 100 MeV constitute, respectively, 69%, 84%,

84%, and 83% of the fluxes ( $3.3 \times 10^{10}$ ,  $9.7 \times 10^9$ ,  $2.9 \times 10^9$ , and  $6.6 \times 10^8 \text{ cm}^{-2}$ ) obtained by integrating over the entire solar cycle.

During several of the time intervals listed in Table 1, more than one proton-emitting flare occurred resulting in discrete interplanetary particle events. Such intervals are identified in Table 1 by the appearance of more than one set of peak flux values. Because such closely spaced solar flares are usually causally related, neither they nor their associated particle fluxes may be considered statistically independent (i.e., random). However, because independence of event occurrence is assumed in the statistical treatment of event occurrence, such occurrences have been grouped in Table 1. Thus the basic unit for this analysis is the event group. (For convenience, such groups will be referred to as events.) Note that with this grouping all statistical nonindependence has not been eliminated, as events separated by months may still be associated with a common long-lived solar-active region. The three periods of early 1969 constitute an example of this. It is not felt that this effect seriously compromises the validity of the analysis, for the distribution of event separations is consistent with the exponential distribution characterizing the separation distribution of truly random events.

The data found in Table 1 results from a variety of sources, mainly associated with the IMP series of spacecraft with geocentric, highly elliptical orbits. The first three periods identified occur before the launch of IMP 4 and are not covered as well as later periods.

For all periods after the launch of IMP 4 (May 24, 1967), proton flux data of C. O. Bostrom (Johns Hopkins University/Applied Physics Laboratory; protons above 10, 30, and 60 MeV), L. J. Lanzerotti (Bell Telephone Laboratories; 10 to 17 and 17 to 19 MeV protons), F. B. McDonald (Goddard Space Flight Center; 8 proton energy channels from 10 to 80 MeV) and J. A. Simpson (University of Chicago; 11 proton energy channels from 10 to 93 MeV) were available to the author for varying periods, either directly from the experimenter or as part of the data base of the National Space Science Data Center. All four data sets were available for the IMP-4 period (May 1967-May 1969). A detailed study of the mutual consistency of these data revealed agreement in event-integrated fluxes typically to better than 25% (King, 1972). As such the data for the IMP-4 period may be considered quite reliable; and given that the IMP-5 experiments were essentially the same as those flown on IMP 4, the data for the IMP-5 period (June 1969-December 1972) may be considered similarly reliable.

The peak fluxes of Table 1 were taken directly from Bostrom's data, while the event integrated fluxes were obtained as follows. All the experimental data were plotted for each event and a smooth spectral curve was drawn. From this curve the integral fluxes above 10, 30, and 60 MeV were determined. These points were then used to estimate an exponential rigidity spectral parameter for use in extrapolating to higher energies. This is  $R_0$  as given in Table 1. The tabulated

fluxes of protons above 100 MeV are as extrapolated using this fit. One should be extremely careful in extrapolating beyond 100 MeV where there is almost no solar-proton flux data available.

There is one very important exception to this. Bazilevskaya et al (1973) have plotted an intensity-time profile for the flux of solar protons above 200 MeV as measured by stratospheric balloon experiments during the large August 1972 events. The area under their published curve has been integrated by the author to obtain an event fluence of  $1.3 \times 10^7 \text{ cm}^{-2}$ . The error introduced by the integration technique is probably less than a factor of 2. Intrinsic errors for their data points are not discussed in their paper. The  $> 30$  and  $> 60$  MeV August 1972 fluences of Table 1 and the  $> 200$  MeV fluence of Bazilevskaya et al are much better fit by an exponential in energy representation with an e- folding energy of 26.5 MeV than by other standard spectral representations (exponential in rigidity, power law in energy). Accordingly, for energy thresholds between 30 and 200 MeV for the August 1972 event,

$$J(>E) = 7.9 \times 10^9 \exp[(30-E)/26.5] \quad (1)$$

with E in MeV and J in  $\text{cm}^{-2}$ . From this representation, the  $J(> 100 \text{ MeV})$  value given in Table 1 is obtained.

Summarizing the energy coverage of this analysis, the August 1972 integral energy spectrum to be used for missions involving the occurrence of anomalously large events is known over 10-200 MeV, while for all other missions the integral energy spectrum to be presented is reliable over 10-100 MeV.

Figure 1, containing event-integrated fluxes of solar protons of energies above 30 MeV, contrasts the solar-cycle-20 proton fluences with those observed during the 19th solar cycle. The solar-cycle-19 data were taken from the compilation of Yucker (1972), which drew on numerous earlier sources. The main points to be noted in Figure 1 are (1) the lull in activity between cycles 19 and 20, (2) the generally more active character of cycle 19 in terms of event-occurrence rate and fluences amplitudes, and (3) the comparability of the August 1972 flux level with the largest cycle-19 event. Relative to the events of November 1960, which have been grouped to give the  $10^{10}/\text{cm}^2$  data point, estimates of the event fluence from various sources have differed by almost a factor of 10. Yucker's list used the most recent and largest value due to Masley. A difference of this magnitude in the largest event of a cycle has a great effect on flux predictions and mission planning. The space community is fortunate in having sufficiently good satellite measurements of proton fluxes for solar cycle 20 so that uncertainties in predictions result from problems inherent in statistical analyses of small numbers of events and not additionally from large uncertainties in cycle-20 fluence values.

### SECTION 3. RELEVANCE OF THE DATA TO THE FUTURE

In order to make statistical predictions about the future two points are important. First, there should be statistical significance



in the data base used; and second, the period for which the predictions are made should be similar to the period during which the data base was accumulated. From Figure 1 it is apparent that if all the events of cycles 19 and 20 were used, a statistically more significant data base is obtained than if only the data of one cycle or the other were used. On the other hand, the greater event-occurrence rate and the generally larger event fluences of cycle 19 demonstrate that cycles 19 and 20 were not statistically similar. From the point of view of Burrell's extension of Poisson statistics (discussed subsequently), the probability that the 19th cycle, with 32 events with  $J(> 30 \text{ MeV}) \geq 5 \times 10^6 \text{ cm}^{-2}$ , and any cycle with as few as 20 such events (as had the 20th) should have arisen from the same governing distribution is only 5%.

The relevant question becomes: What are our expectations for the statistical character of cycle 21? Webber (1967) has shown a general trend for annual-integrated solar proton fluxes to be linearly related to mean annual sunspot numbers. Although this trend is not useful in predicting anomalously large fluxes such as those occurring in 1972 and as such should not be depended upon by mission planners, the largest annual mean sunspot number of a solar cycle is assumed here to be indicative of the general statistical character of that cycle's activity. Figure 2 contains a plot of such sunspot numbers for the last 20 cycles. It is quickly apparent that solar cycle 19 was very extraordinary and that cycle 20 was a very ordinary cycle. Based on

the general structure of Figure 2, it is probable that cycle 21 will be more similar to cycle 20 than to cycle 19. For this reason and due to the previously mentioned greater confidence one has in cycle-20 fluence values, the following analysis is restricted to the use of cycle-20 data in obtaining cycle-21 predictions.

#### SECTION 4. THE ANALYSIS

Let  $F$  be the base-10 logarithm of a fluence (log fluence, for short) associated with all solar protons of energies greater than  $E$  encountered during a space mission of duration  $\tau$ . The probability,  $P$ , of exceeding  $F$  in a similar mission is

$$P(>F, E; \tau) = \sum_{n=1}^{\infty} p(n, \tau; N, T) \times Q(>F, E; n) \quad (2)$$

Here  $p(n, \tau; N, T)$  is the probability of occurrence of  $n$  events over duration  $\tau$ , given that  $N$  events occurred during the one past observation interval of duration  $T$ .  $Q(>F, E; n)$  is the probability that, given the occurrence of  $n$  events, the log of the combined fluence (again, log fluence) due to these  $n$  events will exceed  $F$ .

If  $q(F,E)$  is defined as the probability density (distribution function) for the log fluence,  $F$ , associated with individual events, then

$$Q(>F,E;n) = \int_{-\infty}^{\infty} dx \, q(x,E) \times Q[>\log(10^F - 10^x), E; n-1] \quad (3)$$

where  $Q$  in the integrand is defined as unity if the argument of the log is zero or negative, and as zero if  $x < F$  and  $n=1$  simultaneously.

Since the convolution equation 3 is a recursion relation in  $n$  which permits evaluation of  $Q$  for all  $F$ ,  $E$ , and  $n$ , once  $q(F,E)$  is specified, it is clear that the evaluation of  $P(>F,E; \tau)$  is dependent on the specification of the event-occurrence probability function,  $p(n,\tau;N,T)$  and the one-event log fluence distribution function,  $q(F,E)$ . Note that the specification of  $p$  and of  $q$  constitutes two separate problems which must be independently addressed.

The first problem encountered in the specification of  $q(F,E)$  is the fact that a very large fraction (69-84%) of the cycle-20-integrated fluence occurred during one week in August 1972. It seems eminently reasonable to treat anomalously large (AL) events separately from the large number of more ordinary (OR) events, and this is done and justified in the subsequent analysis.

At some future time (after the passage of several solar cycles statistically similar to that cycle for which mission-fluence estimates are desired) there may be data available on several AL events from which a log fluence distribution function can be given, possibly with

the use of extreme value statistics (see Gumbel, 1954). Alternatively, at some closer point in time, the solar physics community may have come to a sufficiently good understanding of solar flare processes that a fluence distribution may be specified theoretically. However, at this point in time there is but one AL event from a cycle (20) similar to our expectations for cycle 21. As such, no better assumption can be made than that all AL events which occur in cycle 21 will have a spectrum identical to that observed in August 1972.

With the distinction between OR and AL events, and with the assumed commonality of spectrum of all AL events, the basic equations for the probability of exceeding log fluence  $F$  in duration  $\tau$  become

$$P(>F;\tau) = \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} p(k,\tau;N_{AL},T) \times p(n,\tau;N_{OR},T) \times Q\left\{>\log[10^F - (k \times 10^B)];n\right\} \quad (4)$$

$$Q(>F;n) = \int_{-\infty}^{\infty} dx \, q(x) \times Q[>\log(10^F - 10^x);n-1] \quad (5)$$

Here  $k$  and  $n$  index different numbers of AL and OR events,  $q(x)$  is the log fluence distribution function for OR events only,  $B$  is the log fluence for AL events, and  $Q$  in a summand or integrand is defined as unity for zero or negative values of the argument of the log.



In the integrand of equation 5,  $Q$  is defined as zero if  $x < F$  and  $n=1$  simultaneously. Note that the  $E$  dependence has been suppressed; spectral considerations will be made after the analysis is developed for a single energy.

An analytic expression for the OR event log fluence distribution function,  $q(F)$ , must next be selected; and the parameters in the expression must be determined by the appropriate choice of data from Table 1. First of all, as in past analyses, it is assumed that  $F$  is normally distributed:

$$q(F) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left[ - \frac{1}{2} \left( \frac{F - \bar{F}}{\sigma} \right)^2 \right] \quad (6)$$

where  $\bar{F}$  is the mean log fluence and  $\sigma$  is the standard deviation. Such a functional dependence is very useful for analysis and represents the cycle-20 data adequately but not perfectly.

The next question to be addressed is the determination of the parameters  $\bar{F}$  and  $\sigma$  from the data of Table 1. There are 24 OR events listed in Table 1. One may use all of these, only the larger half, or some other fraction. There is nothing more arbitrary in using the larger half rather than all the events since an arbitrary fluence threshold was initially utilized in selecting events for inclusion in Table 1. Table 2 shows the mean log fluences and corresponding standard deviations for four energy thresholds and for different selections of Table 1 events. Note that when  $\bar{F}$  and  $\sigma$  are based on the 12 largest OR events, the AL-event log fluence exceeds  $\bar{F}$  by more than  $4 \sigma$  at



each energy. Note also that the relative difference between the  $\bar{F}$  values determined with and without the AL event (considered as an OR event) is small everywhere and increases with increasing energy. The corresponding relative difference in  $\sigma$  values is much greater.

The difference in mission fluence results when selecting different  $\bar{F}$  and  $\sigma$  from Table 2 will be examined in Section 5. Interestingly, due to the larger standard deviation, the probability of an event fluence exceeding a sufficiently large value is greater when  $\bar{F}$  and  $\sigma$  are determined using all 24 OR events rather than just the 12 largest OR events. For example, for a 10 MeV threshold, the probability of having an event log fluence greater than 9.3 is 4%, using either 12 or 24 OR events. But the probability for exceeding 9.8 is .04% or .5% according to whether 12 or 24 events are used. Presumably, inclusion of the next smaller 24 OR events would result in a yet greater probability for exceeding very large event fluences. This effect is clearly unrealistic, and points to the need for caution in the use of the normal log fluence distribution and the selection of the parameters. Fortunately, at the confidence levels and mission durations of principal interest, log fluence probabilities are almost entirely dependent on AL-event-occurrence probability and only very weakly dependent on OR probabilities, such that no significant errors result from doing the analysis with the  $\bar{F}$  and  $\sigma$  determined from the largest 12 OR events at each energy. This has two advantages: first, as smaller values of  $N_{OR}$  are considered, the sum over  $n$  in equation 4 converges more

rapidly, and less computer time is required to reach a desired accuracy; and second, it seems more likely that large log fluence probability should be more dependent on observed large log fluences than on a combination of observed large and less-large log fluences.

To complete the set of working equations required for this analysis, the probability,  $p$ , of observing exactly  $n$  events in a future interval of duration  $\tau$ , given that  $N$  events were observed in a past interval of duration  $T$ , is given by Burrell (1972) as

$$p(n, \tau; N, T) = \frac{(n+N)!}{n!N!} \times \frac{(\tau/T)^n}{[1+(\tau/T)]^{1+n+N}} \quad (7)$$

The derivation of this equation is briefly explained. Assume that the occurrence of events is random and that events, although individually rare, occur at such a rate that the number of events expected over time periods of interest is not extraordinarily large or small. The occurrences of such events is then describable by Poisson statistics:

$$P(x; \mu) = \mu^x e^{-\mu} / x! \quad (8)$$

$P$  is the probability of observing  $x$  events in unit time, given that the mean or expected number per unit time is  $\mu$ . The parameter  $\mu$  need not be an integer.

It is the point of view of statistical analyses that statistical processes are governed by noumenal distribution functions and that repeated observations yield information on the values of the parameters in any distribution function. That is, although it may be asserted from the randomness of solar proton events that their occurrence should

be governed by Poisson statistics, the governing Poisson distribution function may have any value of the parameter  $\mu$ . A single observation of some number of events over unit time is compatible with any  $\mu$ , although at differing probability levels. Several such observations help to determine which of the infinity of possible Poisson functions is in fact governing the process of interest.

In analyses prior to Burrell's which employed Poisson statistics, the mean occurrence rate (events per year or per day) as observed over one solar cycle (cycle 19) was taken as the parameter  $\mu$  (i.e., as selecting which Poisson function was operative). On the other hand, Burrell took the point of view that the number of events observed over solar cycle 19 was really only one data point from which it is risky to claim which Poisson function is operative. He then reinterpreted the Poisson distribution (equation 8) as giving the probability that the operative distribution is characterized by the parameter  $\mu$ , given one observation of  $x$ . This appears to be reasonable in that the integral of  $P(x;\mu)$  over  $\mu$  from zero to infinity, for any value of  $x$ , is unity.

If  $N$  events were observed in past unit time, then the probability density that the operative Poisson distribution is characterized by  $\mu$  is given by

$$P_1(N;\mu) = \mu^N e^{-\mu} / N! \quad (9)$$

If the operative Poisson distribution is characterized by  $\mu$ , then the probability of observing  $n$  events in future unit time is

$$P_2(n;\mu) = \mu^n e^{-\mu} / n! \quad (10)$$

The probability of observing  $n$  events in future unit time, given the observation of  $N$  events in past unit time, is the probability that a given Poisson distribution is operative times that distribution's probability for  $n$  events, summed (integrated) over the infinity of possibly operative Poisson distributions. That is

$$P(n;N) = \int_0^{\infty} d\mu P_1(N;\mu) P_2(n;\mu) \quad (11)$$

Upon generalization to the case of differing past and future observation times, one obtains equation 7 first given by Burrell.

Quantitatively, the Burrell distribution of equation 7 may be compared to the simple Poisson distribution of equation 8 in which the parameter  $\mu$  is taken directly from one past observation period. The Burrell distribution is broader than the Poisson, with greater probability of observing numbers of events far removed from the expected value and less probability near the expected value. As an example, Figure 3 shows the situation for  $\mu=N=4$  and  $T=\tau=1$ . Using Stirling's formula, the ratio of Poisson to Burrell probabilities for the case  $\tau=T$  and  $n=KN$  ( $n=K\mu$  in Poisson notation) may be written as  $(K+1)^{1/2} [2/(K+1)]^{1+N(K+1)} e^{(K-1)N}$ . At  $n=N$ , the Poisson probability is  $\sqrt{2}$  times greater than the Burrell probability for all  $N$ . At

$n=2N$ , the ratio of Poisson to Burrell probabilities declines from 0.93 to 0.036 as  $N$  increases from 1 to 16, while at  $n=4N$ , this ratio declines from 0.18 to  $2.8 \times 10^{-6}$  as  $N$  increases from 1 to 8. Thus the effect of the use of Burrell statistics instead of conventional Poisson statistics is the calculation of greater probability of exceeding a given (large) mission fluence due to the probability of encountering more events.

Figure 4 illustrates the function  $p(n, \tau; N, T)$  for the cases of 12 observed events ( $N$ ) in the past observation period of seven years ( $T$ ), for several future mission durations ( $\tau$ ) ranging from one month to seven years.

Figure 5 illustrates the probability of exceeding any given number of events for missions of several different durations for the case of one observed event in a past seven-year period. Figure 6 illustrates the probability of occurrence of at least one event during missions of varying length for the cases of one and twelve events observed in a past seven-year period. The  $N=1$  curve of Figure 6 will subsequently be used to specify the region of confidence-level mission-duration space in which ordinary events are negligible.

To summarize the approach, the key equations are 4-7. The key assumptions are (1) the separation of ordinary and anomalously large events, (2) the commonality of spectrum for all anomalously large events, (3) the adoption of a normal distribution for the log fluences of the ordinary events, and the choice of any particular set of past



events for the determination of the parameters in the distribution function and (4) the adoption of Burrell statistics to compute probabilities of event occurrences.

## SECTION 5. RESULTS

There are basically two types of results: (1) those demonstrating the extent of quantitative differences following from different assumptions, and (2) those following from what may be considered the best set of assumptions and which are recommended for use.

Figure 7 is an example of the first type of result. This figure contains plots of the probability of exceeding mission log fluence,  $F$ , in a one-year mission for five different ways of choosing the input data. Curves V and W result from the use of ordinary (OR) and anomalously large (AL) events as described in the preceding section; the difference in the two curves results from the selection of the 12 (V) and 24 (W) largest OR log fluences in the determination of the log fluence distribution function parameters. Curve X results from the total neglect of OR events (i.e., it is assumed that the only cycle-20 activity was the one AL event of August 1972). Curves Y and Z result from the failure to distinguish between OR and AL events; for these curves it was assumed that all events are OR events, describable by a log normal distribution, the parameters of which were obtained by a consideration of the largest 13 (Y) and 25 (Z) event log fluences

(including August 1972). Curves Y and Z are included as a matter of interest and not as viable alternatives to the other curves since there is no justification for including in a distribution an event which contributes two to three times as much fluence as all other events combined. The input parameters for the five curves of Figure 7 are given in the >10 MeV column of Table 2.

In comparing curves V and W, note the slight differences in structure for log fluence  $F \lesssim 10$ . For instance, curve V corresponds to lower probability at small log fluence because, given a smaller  $N_{OR}$  (number of past observed OR events), there is a greater chance of getting no events during the mission. The most important feature of curves V and W is that for log fluences greater than 10, these curves are indistinguishable from each other and from curve X. The stepped nature of these curves at  $F > 10$  results from the discrete probabilities with which various numbers of AL events are exceeded. The indistinguishability of figures V, W, and X illustrates an important principle, adapted from that given by Burrell (1972) in a somewhat different analysis: if in a given mission at least one anomalously large event occurs, it is permissible to neglect the occurrence of ordinary events.

This principle follows from the fact that the common log fluence of the AL events is several (5.3 and 3.5 for curves V and W)  $\sigma$  larger than the average OR log fluence  $\bar{F}$ . Since this condition is true at 30, 60, and 100 MeV as well as at 10 MeV for which Figure 7

was generated (see Table 2), the neglect of OR events for missions during which any AL event occurs is valid throughout this analysis. In the present statistical framework, AL events happen during a mission with some probability. Thus, the foregoing principle must be generalized to: if for a given mission duration and confidence level, at least one anomalously large event occurs, then ordinary events are negligible. As an example, from Figure 6, there is a 10% chance of getting at least one AL event during a 4.5 month mission. So to determine the fluence levels which will not be exceeded with a 90 or 95 or 99 (etc.) % confidence, one need only consider AL events. On the other hand, to determine the (lower) fluence levels which will not be exceeded with a 50 or 75% confidence, one must consider OR event contributions.

Generally the mission planner requires the smallest fluence level for which he can have a  $Q\%$  confidence that the level will not be exceeded during a mission of interest. To use this analysis, he first refers to Figure 6. He locates  $(100-Q)/100$  on the ordinate, and  $\tau$  on the abscissa. Then if this point lies below and to the right of the  $N=1$  line, he is in the confidence-level mission-duration regime of negligibility of ordinary events. In this case, he proceeds to Figure 5 and selects (or interpolates) the curve for his  $\tau$ . He then reads off the smallest number of events whose probability of being exceeded is less than  $(100-Q)/100$ . Finally, he multiplies the August 1972 fluences given in Table 1 by this number of events to obtain his desired result. (Alternatively, the August 1972 spectrum given in equation 1 may be used for energies between 30 and 200 MeV.)

As an example, suppose a mission planner must have, for a one-year mission, fluence levels (vs. energy) which will not be exceeded with a 99% confidence level. The (1-.99),  $\tau=1$  point is in the ordinary-event-negligibility regime of Figure 6, so he proceeds to the  $\tau=1$  year curve of Figure 5. The smallest number of events with a probability-of-being-exceeded less than .01 is two. Then by doubling the August 1972 fluences of Table 1, one may be 99% confident that the mission fluence ( $\text{cm}^{-2}$ ) of protons above 10, 30, 60, and 100 MeV will not exceed  $4.5 \times 10^{10}$ ,  $1.6 \times 10^{10}$ ,  $4.9 \times 10^9$ , and  $1.1 \times 10^9$ .

In the confidence-level mission-duration regime in which OR events are not negligible, the full analysis detailed in Section 4 must be used. With OR event log fluence distribution function parameters based on the occurrence of 12 OR events in a past seven-year observation period (see Table 2, line 1), curves for the probability of exceeding any log fluence less than that associated with one AL event were generated for missions of various durations and for 10, 30, 60, and 100 MeV thresholds. Figure 8 shows the family of such curves for 30 MeV, after conversion from log fluence to fluence.

Comparison of the curves at other energies revealed that the representation

$$G(P, \tau, E) = G^*(P, \tau) g(E) \quad (12)$$

is very accurate, especially above 10 MeV. Here  $G$  and  $G^*$  are fluences,  $P$  the probability that the fluence will exceed  $G$  or  $G^*$ ,  $\tau$  the mission duration, and  $E$  the energy threshold. Figure 8 may be interpreted as

giving  $G^*(P, \tau)$ , with the spectral function  $g(E)$  taking the values 2.22, 1.00, 0.61 and 0.33 at 10, 30, 60, and 100 MeV respectively. Further  $g(E)$  is very well represented over the 30-100 MeV range of threshold energies as

$$g(E) = \exp [0.0158 \times (30-E)] \quad (13)$$

Due to the differences in standard deviations, this separation of the energy dependence introduces maximum error in fluence ( $\leq 50\%$ ) at 10 MeV.

An additional feature of Figure 8 is a set of points denoting the galactic proton fluences to be expected for missions of differing lengths. These will be discussed in the next section.

A set of computer runs was made to compare the results of using 12 24 OR events in determining parameters for short missions. For  $> 30$  MeV protons and for one-month missions, the percent differences in probabilities for exceeding any given fluence level above  $10^7 \text{ cm}^{-2}$  was not greater than 15%. For example, the probabilities of exceeding  $10^8 \text{ cm}^{-2}$  are 8.6% and 7.3%. Thus, even for missions as short as a month, the use of 12 rather than 24 input OR events does not result in significant error.

## SECTION 6. DISCUSSION AND CONCLUSIONS

The solar-proton-fluence data for the major solar events of the 20th solar cycle have been tabulated (see Table 1) and have been utilized in the estimation of mission fluences to be encountered in



space missions of various durations flown in the 1977-1983 time period. The anomalously large event of August 1972 was considered separately from the remaining cycle-20 events. It was shown that if for a given confidence level and mission duration an AL event is expected (as indicated by Figure 6), the ordinary events are negligible. If at least one AL event is expected, Figure 5 is used to determine how many events are expected (see Section 4 for details), and the August 1972 spectrum (see Equation 1) is used to obtain fluences for energy thresholds up to 200 MeV. If no AL event is expected, the fluence of protons above 30 MeV which will be exceeded with probability,  $P$ , for mission duration,  $\tau$ , is plotted vs  $P$  and  $\tau$  in Figure 8. For the same  $P$  and  $\tau$ , the fluence of protons above any other energy between 30 and 100 MeV is obtained by multiplying the fluence of Figure 8 by the spectral function given in equation 13.

Since much of the confidence-level mission-duration plane likely to be of interest corresponds to the occurrence of at least one AL event, it is clear that the most serious deficiency of this analysis lies in the lack of understanding of AL events. Two main questions remain unanswered: does the occurrence of such events depend on the phase of the solar cycle? and what are the distribution functions governing the fluence levels and spectral parameters of such events? It has been assumed in this analysis that the occurrence probability is uniform over the active phase of the solar cycle and that all such events will replicate the August 1972 event in fluence and spectral

characteristics. Further, the spectral function (equation 1) used for this event is greatly influenced by the  $> 200$  MeV point determined by integration of the published intensity time profile determined by U.S.S.R. balloon data. At present, no better assumptions can be made. Because one has no good estimates for the range of fluences at which future anomalously large events may occur, and because the predicted mission fluence depends on the fluences of AL events, one cannot assign reliable error estimates to the results of this analysis.

With respect to the assumption of uniform probability of event occurrence, there has been a suggestion in the cycle 19 and 20 data that anomalously large events are somewhat more likely to occur early or late in the active phase of a solar cycle. Using indirect data, Fritzova-Svestkova and Svestka (1973) have carried this suggestion back to 1942. However, in that the number of past anomalously large events is still very small, it seems unreasonable to consider the point as proven.

It is of interest to contrast the solar proton fluences derived in this analysis with galactic proton fluences. Burrell and Wright (1972) have recently studied galactic particle dosimetry. The galactic proton flux, which must be regarded as a quasi-steady-state component of the interplanetary particle environment, has a value of about  $1.5 \times 10^8/\text{cm}^2\text{-year}$ , independent of energy in the 10-100 MeV threshold range. There is a factor of 2 variation over the solar cycle. The galactic data points of Figure 8 demonstrate that for a two-year mission there is a 75% chance that solar proton fluence ( $E \geq 30$  MeV)

will exceed the galactic fluence, while for a one-month mission, the corresponding figure is only 16%. At higher proton energy thresholds, these percent figures will decrease. The point is that in the limits of short missions and high energies, galactic particle fluence is very important relative to solar particle fluence.

Galactic fluxes are also likely to be of prime importance for solar minimum phases. There are too few solar particle events to construct a reliable solar minimum model at this time. (See Webber, 1966, for compilation of 1961-1965 events.) By 1978, after the current minimum phase, enough data may be on hand to model the mission fluences expected for the 1984-1988 period.

Note that the data used in the analysis were for interplanetary observations taken at a distance of 1 A.U. (earth-sun separation distance) from the sun. As such the predictions must be used for interplanetary, 1 A.U. missions. For spacecraft spending significant amounts of time within the geomagnetosphere, magnetic shielding will decrease the fluence expected for a given confidence level and mission duration. To obtain an estimate of this effect, Stassinopoulos and King (1972) assumed that solar protons are excluded from the magnetosphere at geomagnetic latitudes less than  $63.4^\circ$  ( $L < 5$  earth radii). They computed, for missions with circular orbits, the percent of the interplanetary fluence which would be encountered as a function of orbit altitude and inclination. Figure 9 is taken from their paper.

For planetary or other missions which involve much time spent significantly away from 1 A.U., the heliocentric-distance dependence of event fluences must be allowed for (see also Haffner, 1972). Although spatial characteristics of solar-proton populations as a function of time after a flare are not yet well understood, it is clear that such fluxes are not spherically symmetric. Thus, the observation at one spatial point of an event-integrated spectrum does not permit one to say what spectrum that event would have at another spatial point. However, it seems reasonable to say that on a statistical basis, observations made at the earth's heliolongitude would have been made at any other heliolongitude. The same may not be true of heliolatitude, although this has not yet been empirically tested. If one postulates statistical heliolatitude independence of event fluences, then a suitable helioradial dependence of event fluences is  $r^{-2}$ . This is still a rough estimate in that effects of particle deceleration in interplanetary space are neglected. Consideration of such deceleration would lead to an exponent for a given energy somewhat larger than 2. However, neglect of this effect is probably not significant in view of the assumption made regarding the anomalously large event-fluence distribution. Thus, the mission planner with a mission away from 1 A.U. must compute a mean helioradial distance (average of radial distances equispaced in time) for his mission, say  $r_m$  in AU, and then multiply the fluence level predicted by this analysis (for given confidence level and mission duration) by  $r_m^{-2}$ .

### Acknowledgment

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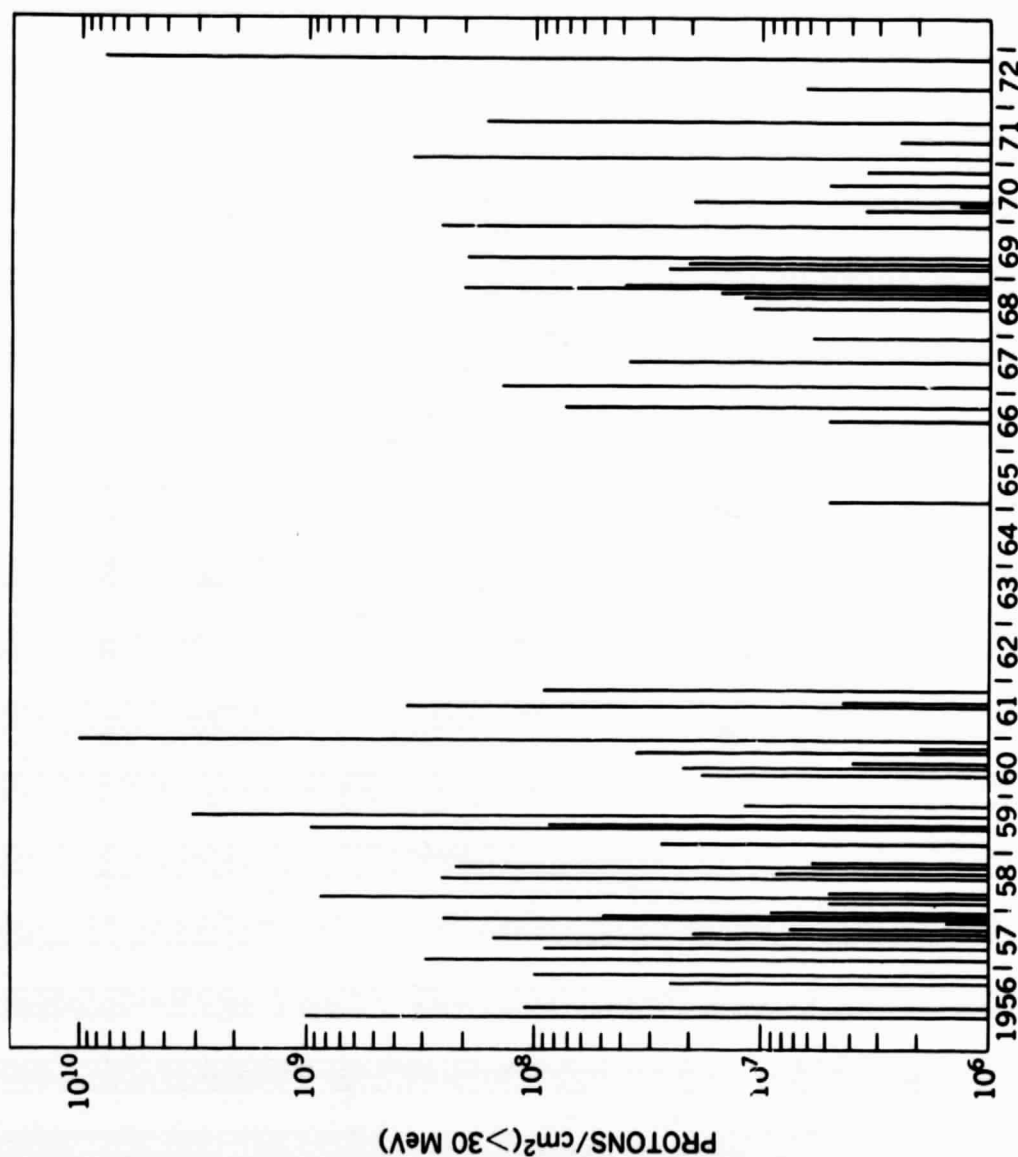


Figure 1. Event-integrated proton fluxes above 30 MeV for the major solar events of the 19th and 20th solar cycles.

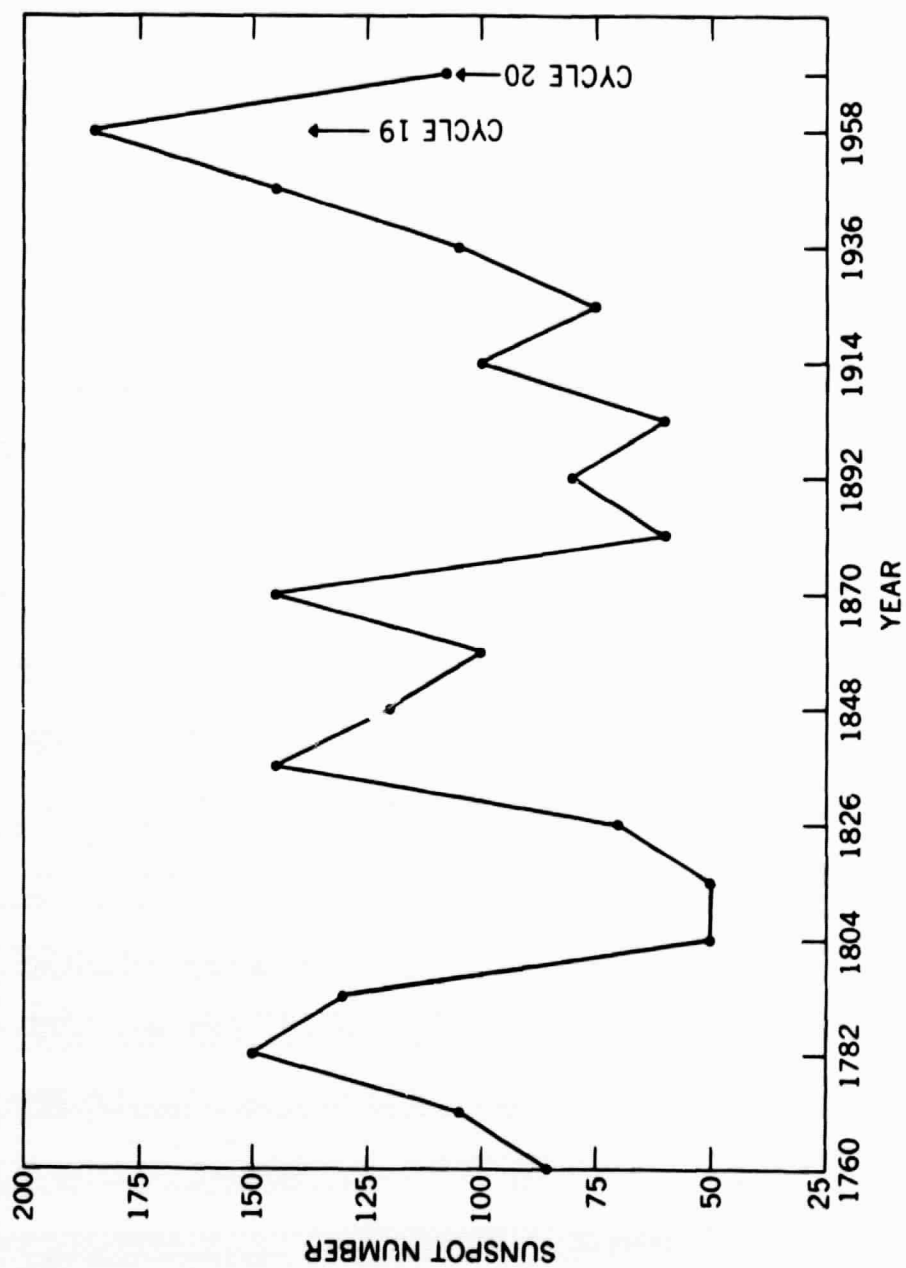


Figure 2. Largest annual-mean sunspot numbers for past 20 solar cycles.

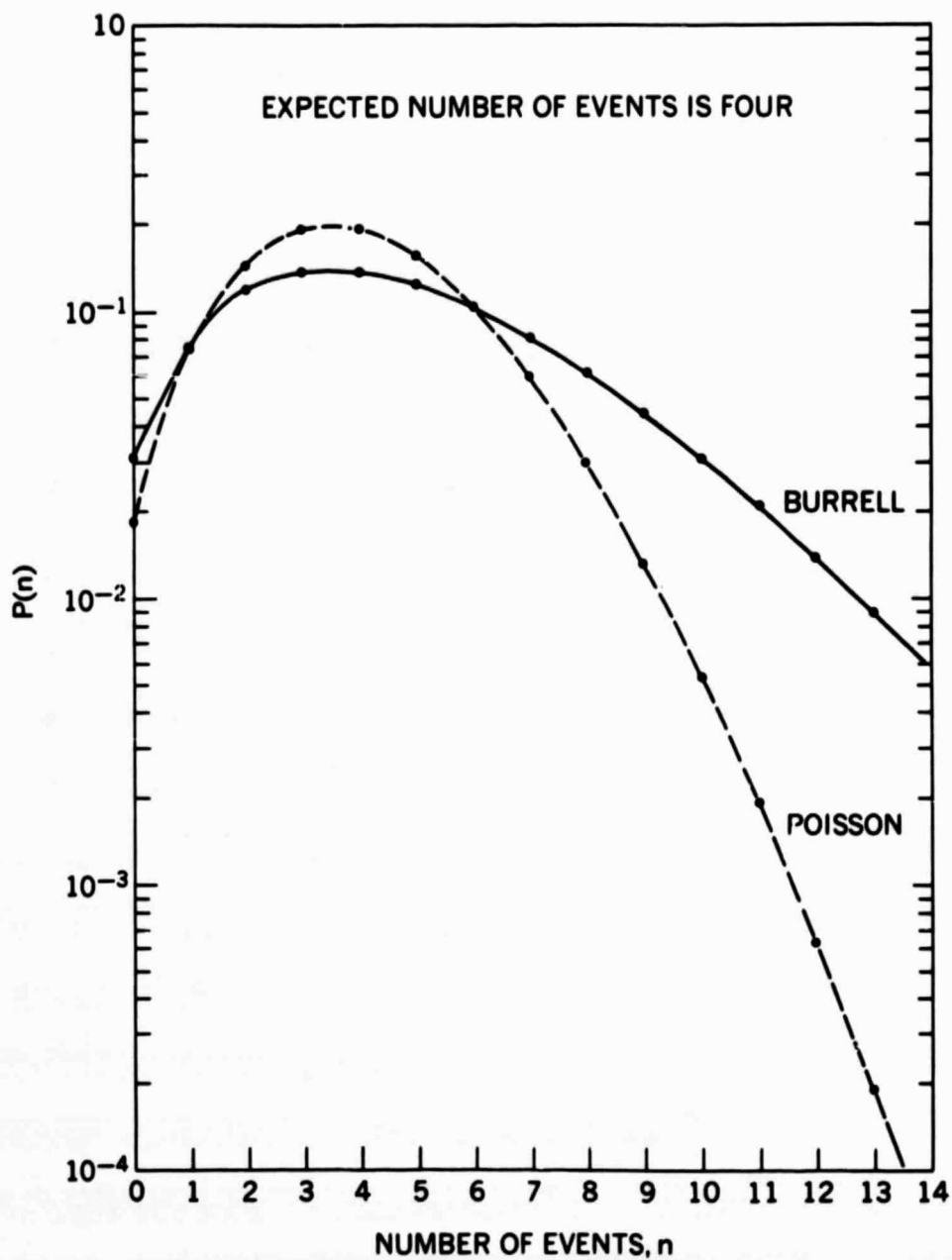


Figure 3. Probability of observing  $n$  events, given that four events are expected, based on Poisson statistics and on Burrell's extension of Poisson statistics.

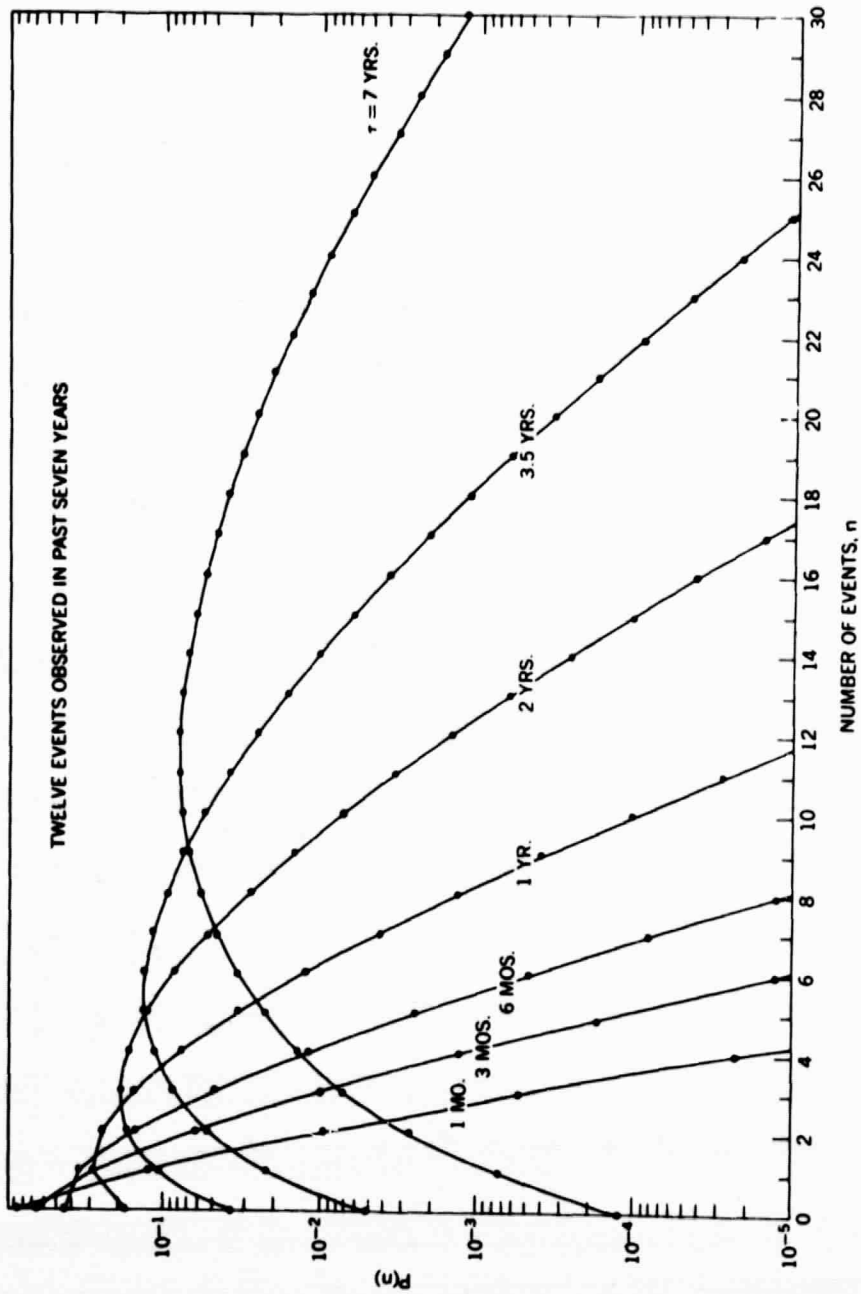


Figure 4. Probability of observing  $n$  events in missions of varying durations given the past observation of twelve events in seven years.



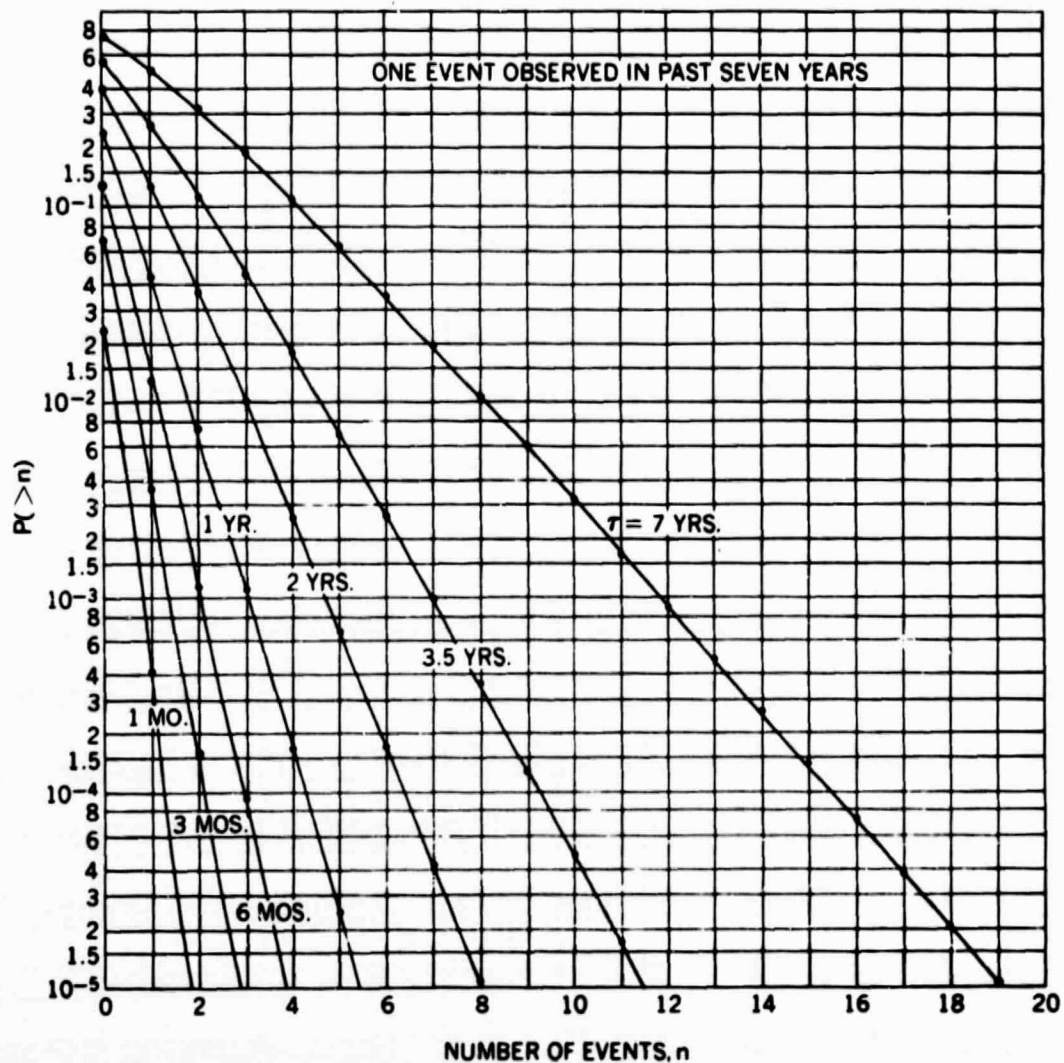


Figure 5. Probability of observing more than  $n$  events in missions of varying durations given the past observation of one event in seven years.

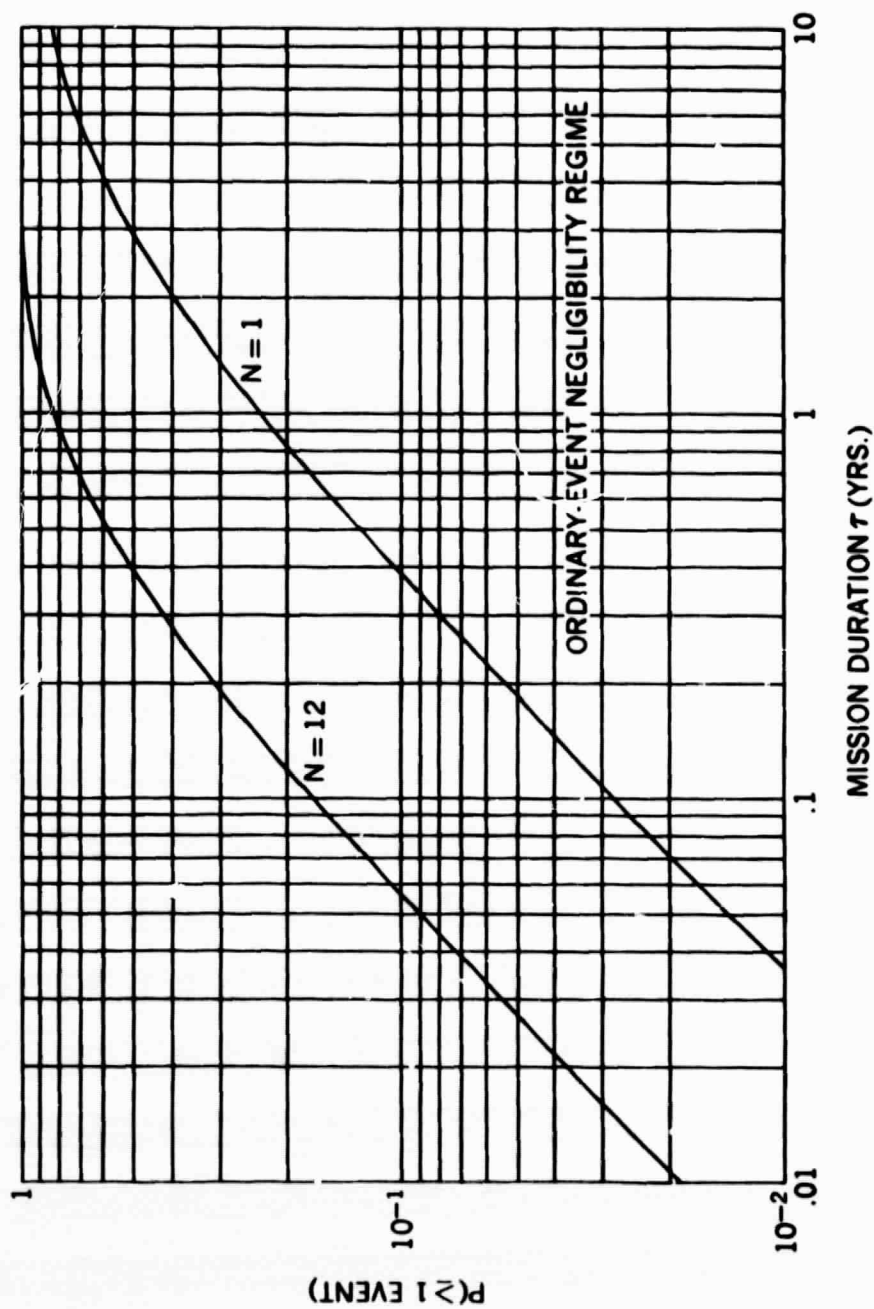


Figure 6. Probability of observing at least one event for missions of duration  $\tau$  given the past observation of one and twelve events in seven years.

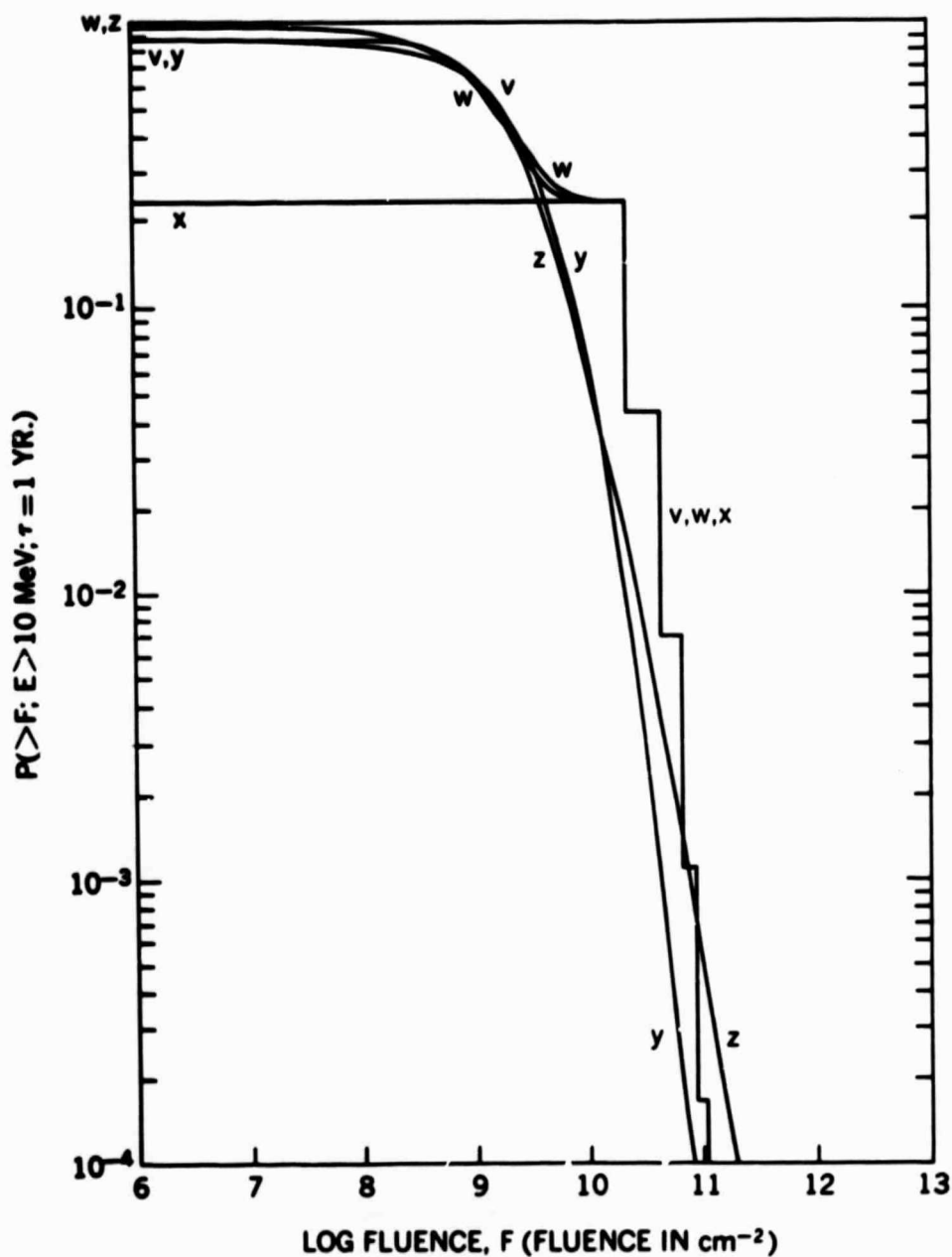


Figure 7. Probability of exceeding log fluences,  $F$ , for proton energy above 10 MeV and for one-year missions given differing ways of handling past events (see text).

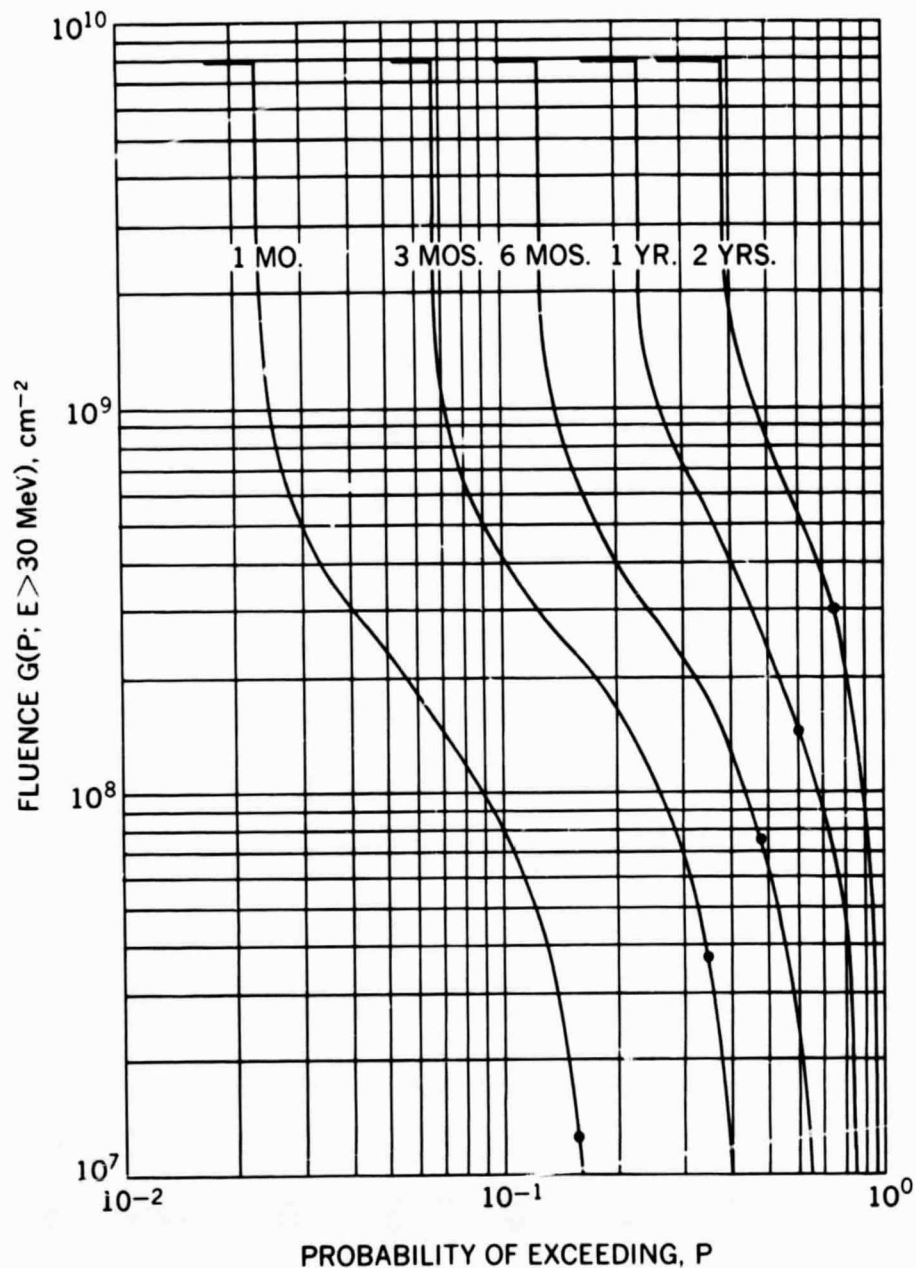


Figure 8. Fluence of protons above 30 MeV which will be exceeded with probability,  $P$ , for missions of varying durations and for fluence levels less than that associated with one AL event. Heavy dots on each curve indicate galactic proton fluence to be encountered.

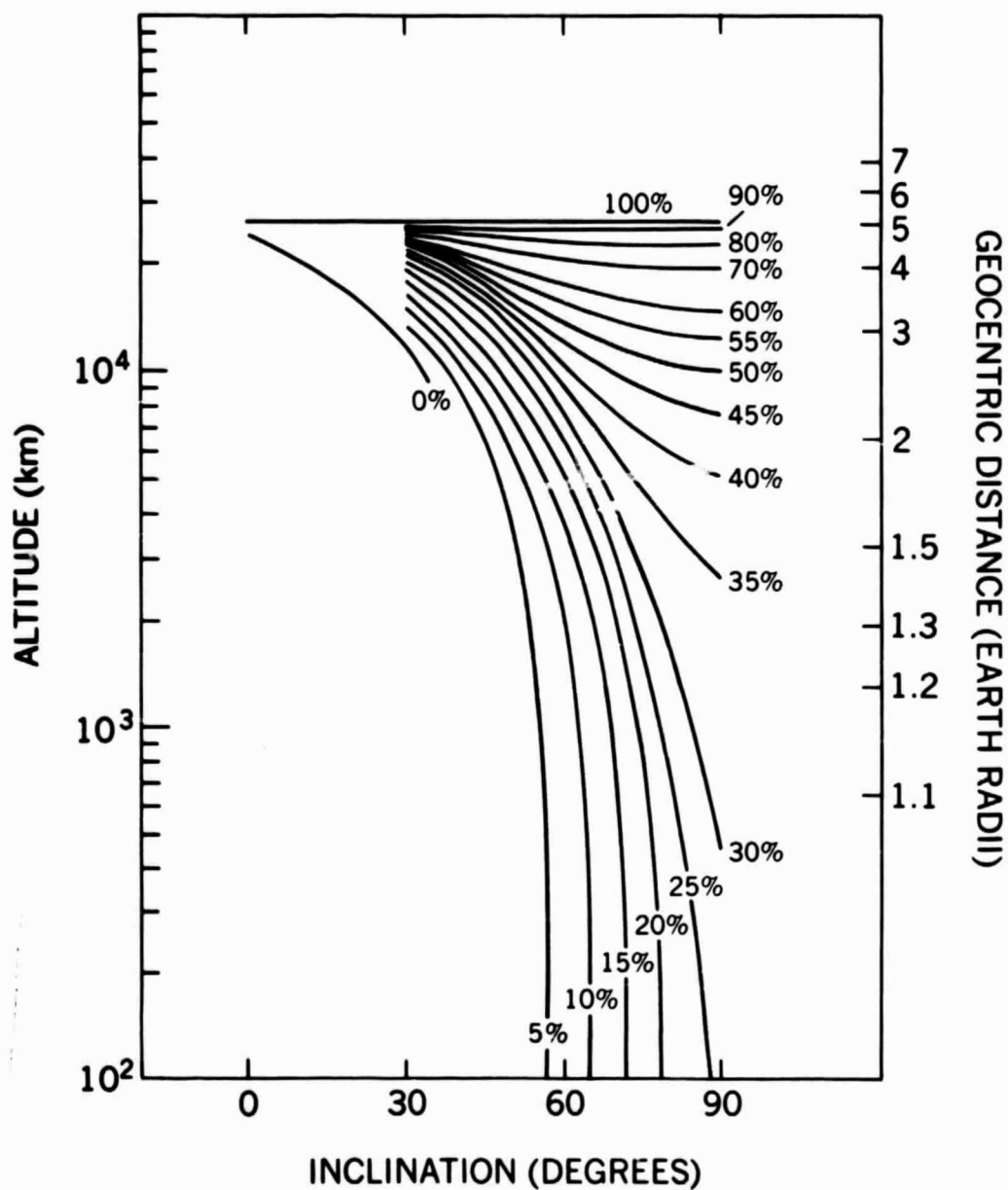


Figure 9. Percentage of interplanetary fluence intercepted by spacecraft in circular geocentric orbits as a function of orbital altitude and inclination.



Table 1. Proton Flux and Spectrum Data for Major Solar Cycle 20 Events

	$J_i(>10 \text{ MeV})$ $\text{cm}^{-2}$	$J_i(>30 \text{ MeV})$ $\text{cm}^{-2}$	$J_i(>60 \text{ MeV})$ $\text{cm}^{-2}$	$J_i(>100 \text{ MeV})$ $\text{cm}^{-2}$	Ro MV	$J_{pk}(>10 \text{ MeV})$ $\text{cm}^{-2}\text{sec}^{-1}$	$J_{pk}(>30 \text{ MeV})$ $\text{cm}^{-2}\text{sec}^{-1}$	$J_{pk}(>60 \text{ MeV})$ $\text{cm}^{-2}\text{sec}^{-1}$
7/07 - 7/09, 1966	$3.8 \times 10^7$	$5.0 \times 10^6$	$1.5 \times 10^6$	$2.3 \times 10^5$	63 <sup>a</sup>			
9/02 - 9/06, 1966	$1.6 \times 10^8$	$8.0 \times 10^7$	$1.3 \times 10^7$	$1.9 \times 10^6$	50			
1/28 - 2/08, 1967	$7.5 \times 10^8$	$1.4 \times 10^8$	$5.0 \times 10^7$	$1.2 \times 10^7$	78 <sup>a</sup>			
5/24 - 5/30, 1967	$6.6 \times 10^8$	$3.8 \times 10^7$	$5.7 \times 10^6$	$4.4 \times 10^5$	43	$1.3 \times 10^4$ $1.4 \times 10^3$	$4.0 \times 10^2$ $3.4 \times 10^2$	$2.9 \times 10^1$ $1.2 \times 10^2$
12/03 - 12/06, 1967	$2.8 \times 10^7$	$5.8 \times 10^6$	$3.1 \times 10^6$	$6.4 \times 10^5$	86	$3.9 \times 10^2$	$1.3 \times 10^2$	$4.8 \times 10^1$
6/09 - 6/11, 1968	$4.1 \times 10^8$	$1.1 \times 10^7$	$1.1 \times 10^6$	$1.0 \times 10^5$	38	$4.4 \times 10^3$	$1.6 \times 10^2$	$6.8 \times 10^1$
9/28 - 10/06, 1968	$8.6 \times 10^7$	$1.2 \times 10^7$	$4.9 \times 10^6$	$9.2 \times 10^5$	70	$4.0 \times 10^2$ $4.5 \times 10^2$	$2.4 \times 10^2$ $7.9 \times 10^1$	$1.3 \times 10^2$ $1.4 \times 10^1$
10/31 - 11/03, 1968	$2.6 \times 10^8$	$1.5 \times 10^7$	$2.5 \times 10^6$	$1.7 \times 10^5$	43	$1.7 \times 10^3$ $1.9 \times 10^3$	$1.3 \times 10^2$ $1.5 \times 10^2$	$1.8 \times 10^1$ $1.4 \times 10^1$
11/18 - 11/21, 1968	$1.1 \times 10^9$	$2.1 \times 10^8$	$7.8 \times 10^7$	$1.3 \times 10^7$	70	$1.1 \times 10^4$	$5.1 \times 10^3$	$1.2 \times 10^3$ <sup>a</sup>
12/04 - 12/09, 1968	$2.8 \times 10^8$	$4.0 \times 10^7$	$7.0 \times 10^6$	$9.6 \times 10^5$	55	$1.9 \times 10^3$	$3.9 \times 10^2$	$6.5 \times 10^1$
2/25 - 3/01, 1969	$6.3 \times 10^7$	$2.6 \times 10^7$	$1.6 \times 10^7$	$7.2 \times 10^6$	159	$1.1 \times 10^3$ $3.5 \times 10^2$	$5.2 \times 10^2$ $1.2 \times 10^2$	$3.0 \times 10^2$ <sup>a</sup> $4.6 \times 10^1$
3/30 - 4/10, 1969	$4.4 \times 10^7$	$1.6 \times 10^7$	$1.0 \times 10^7$	$4.5 \times 10^6$	136	$3.3 \times 10^2$	$1.6 \times 10^2$	$1.1 \times 10^2$ <sup>a</sup>
4/12 - 4/16, 1969	$1.5 \times 10^9$	$2.0 \times 10^8$	$5.7 \times 10^7$	$7.0 \times 10^6$	58	$1.7 \times 10^4$	$1.5 \times 10^3$	$2.0 \times 10^2$
11/02 - 11/06, 1969	$8.7 \times 10^8$	$2.6 \times 10^8$	$1.2 \times 10^8$	$3.2 \times 10^7$	93	$1.6 \times 10^4$	$9.2 \times 10^3$	$2.5 \times 10^3$
1/31 - 2/02, 1970	$2.8 \times 10^7$	$3.4 \times 10^6$	$9.2 \times 10^5$	$4.0 \times 10^5$	84	$3.0 \times 10^2$	$7.8 \times 10^1$	$2.2 \times 10^1$
3/06 - 3/09, 1970	$1.0 \times 10^8$	$1.3 \times 10^6$	$7.8 \times 10^4$	$1.8 \times 10^3$	30	$1.2 \times 10^3$	$1.1 \times 10^1$	--
3/29 - 3/31, 1970	$5.9 \times 10^7$	$2.1 \times 10^7$	$1.2 \times 10^7$	$4.8 \times 10^6$	133	$8.3 \times 10^2$	$2.5 \times 10^2$	$8.1 \times 10^1$
7/23 - 7/25, 1970	$8.1 \times 10^7$	$7.2 \times 10^5$	$3.6 \times 10^4$	$6.0 \times 10^2$	27	$2.6 \times 10^3$	$1.0 \times 10^1$	--
8/14 - 8/17, 1970	$2.6 \times 10^8$	$5.0 \times 10^6$	$4.0 \times 10^5$	$1.2 \times 10^4$	32	$2.3 \times 10^3$	$3.4 \times 10^1$	$3.7 \times 10^0$

a. >500 MeV proton increase observed at Deep River Neutron Monitor.

Table 1 (continued)

	$J_i(>10)$	$J_i(>30)$	$J_i(>60)$	$J_i(>100)$	Ro	$J_{pk}(>10)$	$J_{pk}(>30)$	$J_{pk}(>60)$
11/05 - 11/08, 1970	$9.6 \times 10^7$	$3.5 \times 10^6$	$4.4 \times 10^5$	$4.0 \times 10^4$	45	$5.3 \times 10^2$	$2.1 \times 10^1$	$5.0 \times 10^0$
1/24 - 1/29, 1971	$1.5 \times 10^9$	$3.4 \times 10^8$	$5.9 \times 10^7$	$1.1 \times 10^7$	62	$1.5 \times 10^4$	$5.1 \times 10^3$	$1.1 \times 10^3$
4/06 - 4/08, 1971	$2.9 \times 10^7$	$2.5 \times 10^6$	$3.4 \times 10^5$	$3.3 \times 10^4$	46	$6.4 \times 10^2$	$6.3 \times 10^1$	$1.4 \times 10^1$
9/01 - 9/05, 1971	$3.8 \times 10^8$	$1.6 \times 10^8$	$5.5 \times 10^7$	$2.1 \times 10^7$	103	$4.4 \times 10^3$	$2.0 \times 10^3$	$8.3 \times 10^2$
5/28 - 6/01, 1972	$6.9 \times 10^7$	$6.6 \times 10^6$	$1.5 \times 10^6$	$2.2 \times 10^5$	57	$4.9 \times 10^2$	$3.4 \times 10^1$	$1.5 \times 10^1$
8/04 - 8/09, 1972	$2.25 \times 10^{10}$	$8.1 \times 10^9$	$2.45 \times 10^9$	$5.5 \times 10^8$	$26.5(E_0)$	$1.1 \times 10^6$	$2.6 \times 10^5$	$8.0 \times 10^4$
						$4.4 \times 10^4$	$4.8 \times 10^3$	$8.7 \times 10^2$

a. &gt;500 MeV proton increase observed at Deep River Neutron Monitor.

Table 2. Means and Standard Deviations for Normal Log Fluence Distributions

	>10 MeV	>30 MeV	>60 MeV	>100 MeV
Largest 12 Ordinary (OR) Events	8.81 ± .29	7.92 ± .45	7.41 ± .44	6.78 ± .47
Largest 12 OR Events Plus Anomalous Large (AL) Event	8.93 ± .50	8.07 ± .69	7.56 ± .67	6.94 ± .73
All 24 OR Events	8.27 ± .59	7.28 ± .75	6.63 ± .95	5.77 ± 1.24
All 24 OR Events Plus AL Event	8.35 ± .71	7.39 ± .90	6.74 ± 1.07	5.90 ± 1.36
One AL Event	10.35	9.91	9.39	8.74